Correlations for the Fracture of
Composite Materials

Cambridge Univ. (England)

Jul 81

19960223 115

DEPARTMENT OF DEFENSE
PLASTICS TECHNICAL EVALUATION CENTER
ARRADCOM, DOVER, N. J. 07801
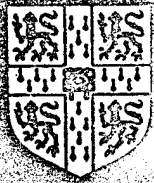
CAMBRIDGE
UNIVERSITY
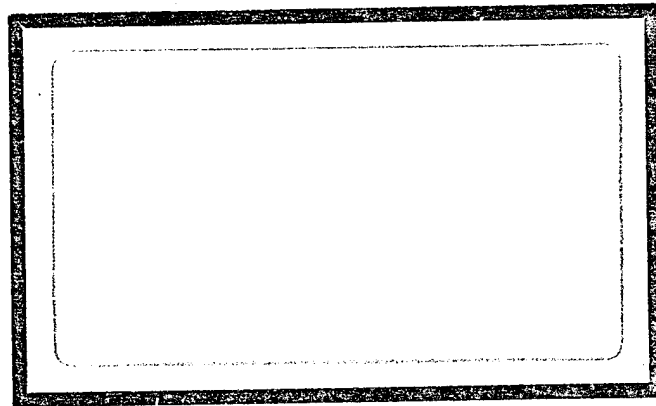
Engineering Department

DISCLAIMER NOTICE
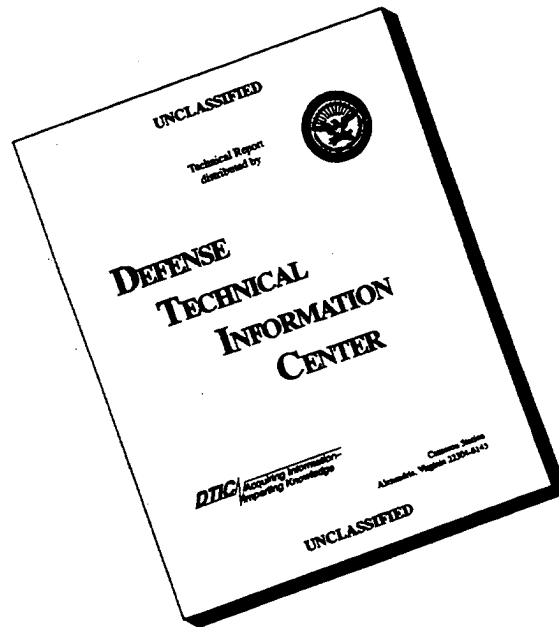
THIS DOCUMENT IS BEST
QUALITY AVAILABLE. THE
COPY FURNISHED TO DTIC
CONTAINED A SIGNIFICANT
NUMBER OF PAGES WHICH DO
NOT REPRODUCE LEGIBLY.

A number of existing theories for both the notched and unnotched strength of fibre composite materials are discussed. These are compared with a large body of existing data which is presented in the form of 'master plots'. These plots show the similarity of behavior of carbon, glass and Kevlar laminates of a variety of constructions, and also identify two distinct failure regimes.

i

CORRELATIONS FOR THE
FRACTURE OF COMPOSITE MATERIALS

J.K. Wells and P.W.R. Beaumont

Cambridge University
Engineering Department
Trumpington Street
Cambridge   CB2 1PZ

ABSTRACT

A number of existing theories for both the notched and un-
notched strength of fibre composite materials are discussed.
These are compared with a large body of existing data which is
presented in the form of 'master plots'. These plots show the
similarity of behaviour of carbon, glass and Kevlar laminates of
a variety of constructions, and also identify two distinct failure
regimes.

1. INTRODUCTION

It has long been known that materials with some ductility show two fracture modes, depending on the length of any notch they may contain. When the notch is short, failure occurs by *general damage* leading to ductile fracture of the remaining section, at a net section stress which is independent of the notch length, a. Longer notches, however, produce crack-tip stresses that are large enough to cause failure at the tip, so that the crack propagates while the deformation in the material remote from it is still elastic, and at an applied stress which depends on the square root of the reciprocal of the crack length, $1/\sqrt{a}$. We call this the regime of *single crack propagation*. The overall behaviour is summarised in Fig. 1: roughly speaking, the material fails at the lower of the two stresses.

Similar regimes may be identified in composite materials. When notches are absent or are short, general damage occurs as dispersed, small cracks in the matrix, as fibre breakage, or as decohesion at the fibre-matrix interface, leading to a degradation and failure of the entire section. But when a long notch is present, it may propagate as a single crack, with little deformation in the material remote from its tip.

2. GENERAL DAMAGE MECHANICS

Fibre composites frequently contain manufacturing defects (such as fibre breaks and ends) and weak fibres which break at low applied loads. *General damage mechanics* is concerned with the formation and aggregation of such dispersed defects leading to failure.

The "quantum" of damage is one broken fibre. Where a fibre has broken, the axial load, formerly carried by the fibre, is carried by shear of the matrix and is thereby transferred to the adjacent fibres. Rosen (1964), Daniels (1945) and Gücer and Gurland (1962) assume the load to be uniformly shared between all other fibres in the section. This assumption, known as the "Equal Load Sharing Rule" (ELS) takes no account of the greater matrix shear near the broken fibre.

A more refined model, known as the "Local Load Sharing Rule" (LLS), has been developed by other workers including Hedgepeth (1967), Hedgepeth and Van Dyke (1967), Scop and Argon (1969) and Zweben (1968) and (1974a). Hedgepeth found that the load on a fibre adjacent to i broken fibres was increased by a factor k_i (the "load concentration factor") above the average fibre stress in the composite. A two-dimensional plastic matrix analysis suggests that the factors are given by:

$$k_i = \frac{4.6.8 \ldots \ldots (2i+2)}{3.5.7 \ldots \ldots (2i+1)}$$

Support for this approach appears in the results of Zweben (1968), who used it to predict the ultimate strength of model glass/epoxy composites and concluded that the failure occurred when two adjacent fibres broke (that is, when $i = 2$). The approach was refined further by Scop and Argon (1969), who examined (numerically) the various sequences of fibre failure which can occur in a general-damage failure. They concluded that failure was governed by the first fracture of two fibres adjacent to a broken fibre (when $i = 3$). The predicted strength according to Scop and Argon is given (for a typical distribution of fibre strengths) by the simplified form due to Harlow and Phoenix (1980):

$$\sigma_u \approx \frac{\sigma_o}{k_3}$$

where $k_3 = 1.92$ for an elastic matrix

                  $k_3 = 1.36$ for an elastic/plastic matrix

and σ_o is the "Rule of Mixtures" composite strength

In later work, Argon (1974) studied the effect of specimen size on material strength, finding a size for which the strength was a maximum. For such a sample, Argon predicted that fracture of 2 or 3 adjacent fibres would cause failure, in agreement with his earlier result and the empirical finding of Zweben. The ratio of unnotched to rule of mixtures strength was found to be about 1.3.

3.   <u>SINGLE CRACK MECHANICS</u>

Tests on composite panels with circular holes in them show a dependence of tensile strength on the diameter of the hole, a result which cannot be explained by the classical stress concentration factor. This and other effects of holes and notches have led a number of authors to analyse failure of notched laminates using the methods of fracture mechanics, although the validity of doing so has been questioned (Nuismer and Whitney, 1975).

Waddoups et al. (1971) modelled the circular hole as having two slits extending symmetrically from either side, perpendicular to the load direction. The cracks, known as 'intense energy regions' were

analysed using the Bowie crack solution. Knowing the critical stress intensity factor K_Q, from a fracture test, and the length of the intense energy region, it is possible to calculate the notched strength of the laminate. Reasonable quantitative agreement of this "inherent flaw model" was found with available data.

An alternative approach (the "Point Stress" approach) considers the stress distribution around the crack tip. Whitney and Nuismer (1974) assumed that failure occurs when the stress at some distance, d_o, ahead of the hole, first reaches the unnotched tensile strength of the material, σ_u. For a circular hole of radius R, it can be shown that the failure stress, σ, is given by:

$$\frac{\sigma}{\sigma_u} = \frac{2}{(2 + \alpha^2 + 3\alpha^4)} \quad \text{where} \quad \alpha = \frac{R}{(R + d_o)}$$

For large holes, the classical result of $\sigma/\sigma_u = \frac{1}{3}$ is obtained, whilst for small holes the ratio tends to unity. For a central slit of length $2c$, the notched strength is:

$$\frac{\sigma}{\sigma_u} = \sqrt{1 - \beta^2} \quad \text{where} \quad \beta = \frac{c}{c + d_o}$$

A third approach (the "Average Stress" approach) again places emphasis on the stress distribution ahead of the crack. However,

failure is assumed to occur when the average stress over a fixed distance, a_o, in front of the hole reaches the unnotched strength of the material. For a circular hole, the notched strength is given by:

$$\frac{\sigma}{\sigma_u} = \frac{2(1 - \alpha_2)}{(2 - \alpha_2^2 - \alpha_4^4)} \quad \text{where} \quad \alpha_2 = \frac{R}{R + a_o}$$

and for a central slit:

$$\frac{\sigma}{\sigma_u} = \sqrt{\frac{a_o \, \beta_2}{c}} \quad \text{where} \quad \beta_2 = \frac{c}{2c + a_o}$$

Clearly both the point stress and the average stress criteria rely on the parameters a_o and d_o being independent of hole size in at least a particular laminate construction of a given material system. Whitney and Nuismer (1974) have found experimental evidence to suggest that these parameters are constant for various lay-ups of several different fibre reinforced plastic systems. The values found to give good agreement are:

$$a_o = 3.8 \text{ mm}$$

$$d_o = 1.02 \text{ mm}$$

The Inherent Flaw model also gave fairly good agreement if
the length of the intense energy region was assumed to be 2.03 mm.

It should be pointed out that the use of laminated plate theory
to predict stresses ahead of the notch is questionable, in view of
the known problems caused by free edge effects, matrix crazing and
material non-linearity. However, the stress-field perturbations
caused by these effects may be restricted to a region which is small
by comparison with either a_o and d_o.

4. MASTER PLOTS

The literature contains a large amount of data for fracture strength
as a function of crack length a. These data have been used to con-
struct "master plots" to show the underlying similarities between dif-
ferent materials and lay-ups. To do so, the notched strength data have
been normalised using the unnotched strength quoted by the authors, and
plotted as a function of crack length. The plots (Figs. 2, 3 and 4) show
data for carbon, glass and Kevlar 49 fibres in epoxy or polyester matrices.
Only lay-up sequences which include at least one 0° ply are included; a total
of 250 results for 14 different constructions and three fibre types are in-
cluded; and each result is itself the average of several tests*. Test geo-
metries include central circular holes, central slits and inclined slits.
The crack length a is taken as half the width of a central hole or slit
and to half the projected crack length normal to the loading direction
for an inclined slit. Specimen widths varied between 12 and 70 mm.

---

*A rather similar procedure was used by Whitney and Nuismer (1974)
 to estimate the parameters d_o and a_o for the "point stress"
 and "average stress" criteria, but it encompassed far less data.

The path of the propagating crack varied depending on material type and construction; in general the failure involves splitting parallel to the fibres, and the fracture surface was not always perpendicular to the applied load.

The failure stresses quoted in most papers have already been corrected for the finite width specimens. Where this had not been done, the standard corrections were made using:

$$\sigma = \frac{Y}{\sqrt{\pi}} \, \sigma_{\text{finite width}}$$

where $Y = 1.77 + 0.227 \left(\frac{2a}{w}\right) - 0.510 \left(\frac{2a}{w}\right)^2 + 2.7 \left(\frac{2a}{w}\right)^3$

for a central slit length $2a$, in a specimen width w (Brown and Srawley (1966)). (This correction applies strictly to isotropic materials only, but Nuismer and Whitney (1972) show that a rigorous orthotropic correction factor seems to over-compensate. In any event, the correction is small for almost all the data.)

Figs. 2, 3 and 4 show a pattern of behaviour like that described in the Introduction: fracture by general damage when cracks are short, and by single crack propagation when they are long. When failure is caused by general damage, it occurs when the net section stress exceeds the strength of the material, so that:

8

$$\frac{\sigma}{\sigma_u} = 1 - \frac{a}{w}$$

Generally w is much larger than a, so that the term a/w can
be dropped. This failure criterion is plotted as the horizontal
line $\sigma/\sigma_u = 1$ on the figures.

When, instead, failure is by single crack propagation, it occurs
at the applied stress:

$$\frac{\sigma}{\sigma_u} = \frac{K_{IC}}{\sigma_u \sqrt{\pi a}}$$

(where, again, we assume a/w << 1). This criterion is plotted for
two values of K_{IC}/σ_u which differ by a factor of 2, on the three
figures. The two lines bracket more than 90 % of all the data, for
all lay-ups, fibre types and matrices.

The scatter in the results reflects the inevitable differences
in material preparation and test methods between authors, for which
the normalisation does not allow. Any systematic difference in beha-
viour between holes, slits and angled slits is small by comparison
with this scatter. However, it is noted that |0/+45/90| type lami-
nates have a notched strength which is about 12 % higher than |0/+45|
and other constructions, in the single crack regime. The lower bound
of $K_{IC}/\sigma_u = 0.045$ forms a convenient (conservative estimate for the
single crack mode notched strength of all the laminates considered in
this work.

The fracture data may also be presented in the form of a normalised apparent fracture toughness, K_{IC}/σ_u given by:

$$\frac{K_{IC}}{\sigma_u} = \frac{\sigma}{\sigma_u} \sqrt{\pi a}$$

Figure 5 shows the dependence of normalised fracture toughness on crack length for carbon, glass and Kevlar composites. Single crack mode failures have a constant K_{IC} and appear as horizontal lines, whilst the general damage mode gives a rising failure toughness with crack length. The figures also show the predictions of the point stress and average stress criteria for central slits. For cracks of less than 2 mm length, the data shows a rising trend consistent with the constant stress failure mode, at longer lengths the fracture toughness levels out to a roughly constant value.

The point stress and average stress criteria successfully predict these trends, though values of $a_0 = 1.5$ mm and $d_0 = 0.4$ mm would provide a better lower bound of notched strength for the data reported here. The main feature of these criteria is that, by choosing a *fixed* critical distance (d_0 or a_0) they lead to an apparent fracture toughness which falls when the crack length is shorter than the critical distance. Our interpretation is a different one: that the reduction is due to a change from a single-crack mode of failure to one associated with general damage.

Relationship between Rule-of-Mixtures and Unnotched Strengths

By neglecting any strength contribution from the matrix in |0/90| type laminates an estimate of the ratio of unnotched to Rule-of-Mixtures strength may be made. Data of Nuismer and Whitney (1975) and McGarry and Mandell (1972) suggests that the material only attains about 80 % of its "ideal" strength. This figure is in broad agreement with the early estimate of 74 % by Scop and Argon (elastic/plastic matrix case), but rather lower than Argon's later estimate of about 130 %.

Predictions of Notched Strength using Load Concentration Factors

Work by Zweben (1971) suggested that the notched strength of a laminate could be related to the load concentration factors discussed in Section 2. He argued that if the average fibre stress in the composite is σ, then the stress in the fibre at the root of a notch cutting i fibres is $k_i \sigma$. When this exceeds the ultimate fibre strength the notch-tip will fail, and since $k_{i+1} > k_i$ all subsequent fibres will fail. The notched strength is therefore given by $\sigma = \sigma_f V_f/k_i$ where V_f is the fibre volume fraction of the composite and $i \simeq (V_f/r)^{\frac{1}{2}} a \times$ fibre radius. Zweben pointed out that σ was roughly proportional to the square root of the reciprocal of the notch length $\sqrt{a}$, and commented that the agreement between this and the fracture mechanics result was coincidental. However, this dependence is one of the fundamental results of fracture mechanics, and Hedgepeth and Zweben's results form a separate, but equivalent derivation of this result. In later work Zweben (1974a) considered the effects of matrix plasticity in a new analysis which we applied to unidirectional carbon/epoxy material. Figure 6 compares these results with the data presented earlier, assuming the unnotched strength of the material

to be 80 % of the rule-of-mixtures strength. Both methods are very conservative, though the plastic matrix analysis much less so than the simple model.

Estimate of the Toughness of the Constituent Laminae

The actual path of a crack through a laminate is generally complex; angle-plies often split parallel to the fibres and cross-plies almost invariably do so. Zero degree plies may also split, but all fibres in such layers must fracture at laminate failure. Fibre failure involves processes which absorb much more energy than does splitting, and it is therefore likely that most of the toughness of the laminates considered in this work derives from the 0° plies.

The critical strain energy release rate, G_{IC}, has been found for two constructions using the relation between K_{IC} and G_{IC} for anisotropic bodies derived by Sih, Paris and Irwin, (1965) see Table 1. Typical values of material properties for E-glass/epoxy have been used, and it is assumed $K_{IC}/\sigma_u = 0.07$. It appears that, when the number of aligned plies is allowed for, the energy absorption is principally due to the 0° plies; for such plies $G_{IC} \sim 85$ kJ/m^2.

TABLE 1

Lay-up	K_{IC}^2/G_{IC} (GPa)	σ_u (MPa)	G_{IC} (kJ/m^2)	G_{IC} of 0° ply* (kJ/m^2)
$[0/\pm45/90]_s$	15.6	250	19.7	78.7
$[0/90/0/90]_s$	11.7	333	46.6	93.2

*The toughness of the 0° ply is given by the approximation:

$$G_{IC} (0^o \text{ ply}) = G_{IC} \frac{\text{Total number of plies in laminate}}{\text{Number of } 0^o \text{ plies in laminate}}$$

CONCLUSION

When data for the normalised fracture strength of composites, σ/σ_u, (where σ is the fracture strength of a sample containing a crack of length a and σ_u is the unnotched strength) are plotted against crack length, two regimes of behaviour appear. When cracks are short ($a < 2$ mm), the composite fails by a general-damage mechanism, at a stress which is independent of the initial crack length. But when cracks are longer ($a > 2$ mm), the composite fails by the propagation of the crack, with little or no general damage, and at a stress which depends on $a^{-\frac{1}{2}}$.

General damage consists in the accumulation of broken fibres, until the probability of adjacent breaks reaches a critical level, when failure occurs. The data are in good agreement with Scop and Argon's (1969) estimate of this stress. Single crack failure occurs when the crack length is great enough to focus all damage at the crack tip: then the crack becomes unstable at a stress which can be calculated using the standard methods of fracture mechanics. The data can be plotted to determine the fracture toughness of the composites in this regime. The results show a remarkable consistency: for all the composites for which we could find data, K_{IC}/σ_u lay in the range:

$$\frac{K_{IC}}{\sigma_u} = 0.07 \pm 0.02 \ m^{\frac{1}{2}}$$

Acknowledgement

References

Argon, A.S. (1974) "Statistical aspects of fracture", Composite Materials: Fracture & Fatigue (L.J. Broutman & R.H. Knock, ed.), Academic Press, New York, p.153.

Brown, W.F. and Srawley, J.E. (1966 "Plane strain crack toughness testinf of high strength metallic materials", ASTM STP 410.

Cruse, T.A. (1973) "Tensile strength of notched composites", J. Comp. Mater. 7, 218.

Daniels, H.E. (1945) "The statistical theory of the strength of bundles of threads", Proc. Roy. Soc. 183A, 405.

Glicer, D.E. and Gurland, J. (1962) "Comparison of the statistics of two fracture modes", J. Mech. Phys. Sol. 10, 365.

Harlow, D.G. and Phoenix, S.L. (1980) "The chain of bundles probability model for the strength of fibrous materials I: Analysis and conjectures", J. Comp. Mater. 12, 195.

Hart, W.G.J. (1979) "Residual strength of multiply carbon/epoxy and Kevlar-carbon/epoxy notched laminates" NLR MP79016V, National Aerospace Laboratory, NLR, Amsterdam.

Hedgepeth, J.M. (1961) "Stress concentrations in filamentary structures", NASA TN D882, Langley Research Center.

Hedgepeth, J.M. and Van Dyke, P. (1967) "Local stress concentrations in imperfect filamentary composite materials", J. Comp. Mater. 1, 294.

Mar, J.W. and Lin, K.Y. (1979) "Characterization of splitting process in graphite/epoxy composites", J. Comp . Mater. 13, 278.

Morris, D.H. and Hahn, H.T. (1977a) "Fracture resistance characterisation of graphite/epoxy composites", ASTM STP 617, p.5.

Morris, D.H. and Hahn, H.T. (1977b) "Mixed mode fracture of graphite/epoxy composites: Fracture strength", J. Comp. Mater. 11, 124.

McGarry, F.J. and Mandell, J.F. (1972) "Fracture toughness studies of fibre reinforced plastic laminates", Discussion of the Faraday Section, Chemical Society. University of Nottingham, September 1972.

Nuismer, R.J. and Whitney, J.M. (1975) "Uniaxial failure of composite laminates containing stress concentrations", ASTM STP 593, p.117.

Owen, M.J. and Cann, R.H. (1979) "Fracture toughness and crack growth measurements in GRP", J. Mater. Sci. 14, 1982.

Potter, R.T. (1978) "On the mechanism of tensile failure in notched fibre reinforced materials", Proc. Roy. Soc. Lond. A361, 325.

Rosen, B.W. (1964) "Tensile failure of fibrous composites", AIAA Journal 2, 1985.

Scop, R.M. and Argon, A.S. (1969) "Statistical theory of strength of laminated composites II", J. Comp. Mater. 3, 30.

Sih, G.C., Paris, P.C. and Irwin, G.R. (1965) "On cracks in rectilinearly anisotropic bodies", International Journal Fract. Mech. 1, 189.

Waddoup, M.E., Eisenmann, J.R. and Kaminski, B.E. (1971) "Microscopic fracture mechanics", J. Comp. Mater. 5, 446.

Whitney, J.M. and Nuismer, R.J. (1974) "Stress fracture criteria for laminated composites containing stress concentrations", J. Comp. Mater. 8, 253.

Whitney, J.M. and Kim, R.Y. (1976) "High temperature tensile strength of graphite/epoxy laminates containing circular holes", J. Comp. Mater. 10, 319.

Whitney, J.M. and Kim, R.Y. (1977) "Effect of stacking sequence on the notched strength of laminated composites", ASTM STP 617, p.229.

Zweben, C. (1968) "Tensile failure of fibre composites", AIAA Journal 6, 2325.

Zweben, C. (1971) "On the strength of notched composites", J. Mech. Phys. Solids 19, 103.

Zweben, C. (1974a) "An approximate method of analysis for notched unidirectional composites", Eng. Fract. Mech. 6, 1.

Zweben, C. (1974b) "Fracture of Kevlar, E-glass and graphite composites", presented at ASTM Symposium on Fracture Mechanics in Composites, Gaithersburg, Maryland.
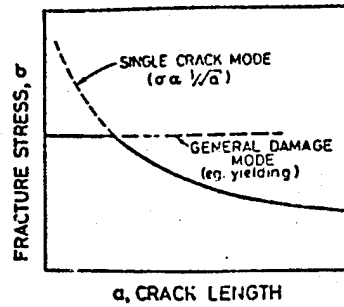
Fig. 1. Schematic showing the stress/crack length dependence
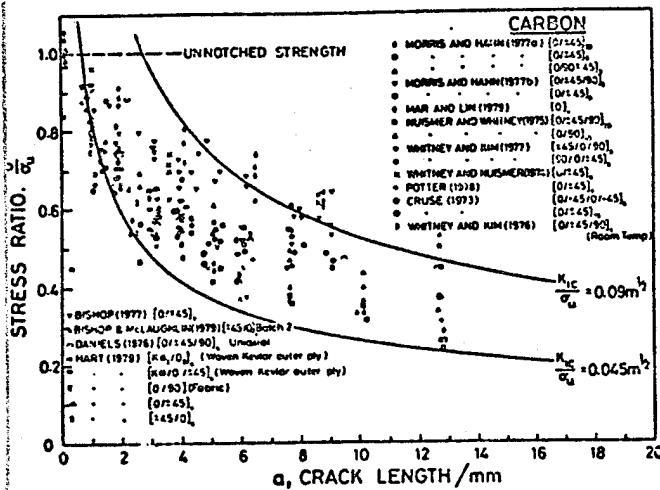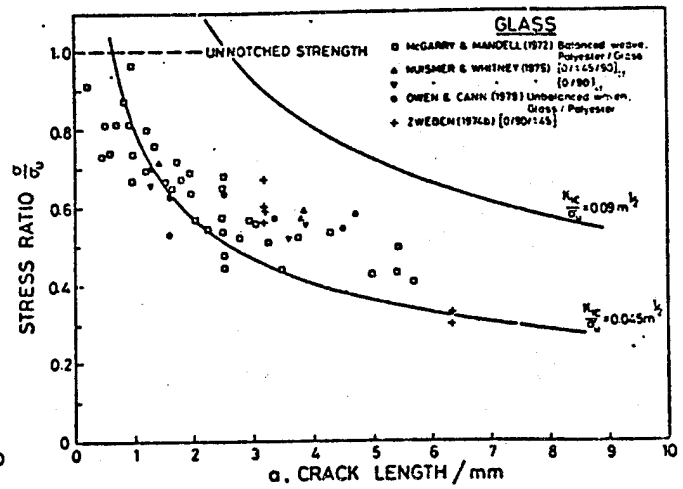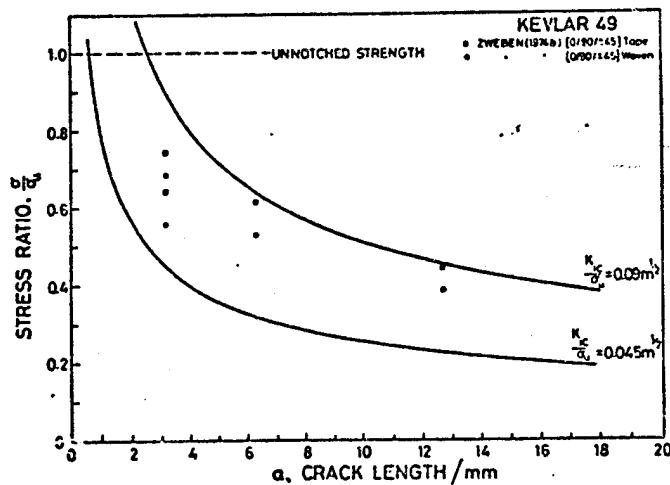of single crack and general damage failures.

Fig. 2

Fig. 3

Fig. 4

Figs. 2, 3 and 4. Variation of stress ratio σ/σ_u with crack
length for carbon, glass and Kevlar composites.

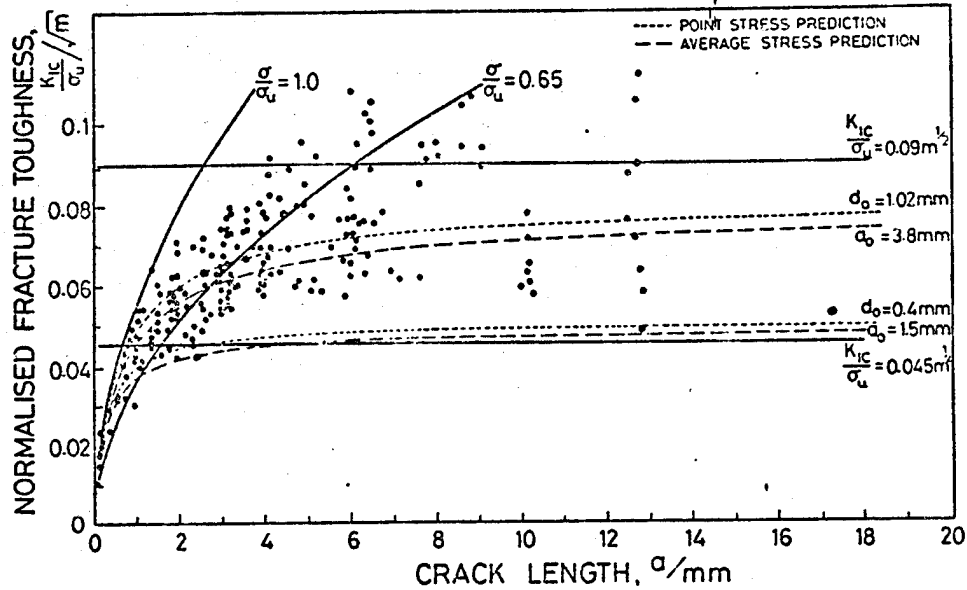Fig. 5. Variation of normalised fracture toughness, K_{IC}/σ_u, with crack
length for carbon, glass and Kevlar composite The Point stress
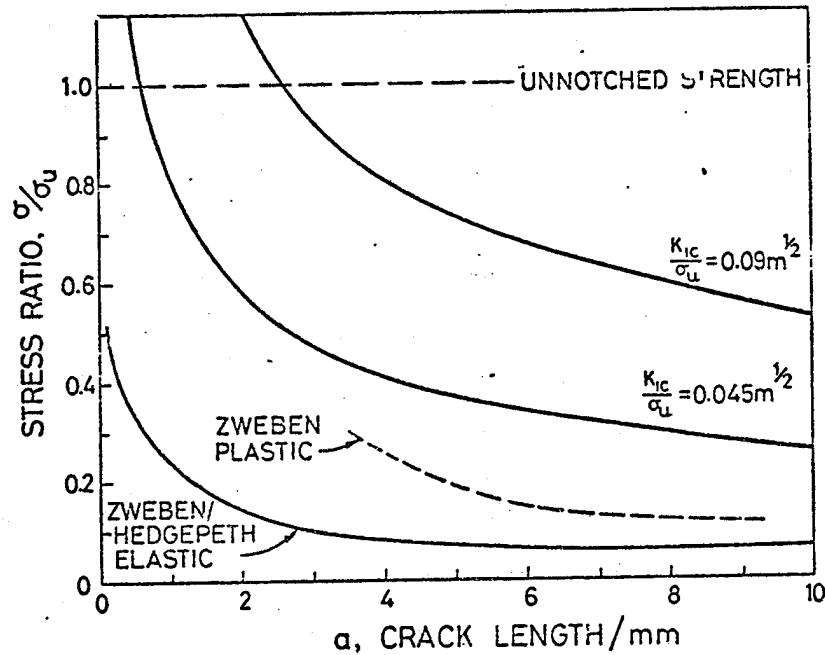and Average stress predictions are also shown.

Fig. 6. The stress ratio variation with crack length predicted
by load concentration factor techniques for elastic and
plastic matrices. The upper and lower bounds of experi-
mental data are shown for comparison.